For all questions, answer choice (E) NOTA means that none of the given answers is correct. Assume that all diagrams are not to scale. Good Luck!

1. Welcome to the 2022 James S. Rickards Fall Invitational's Pre-Algebra Individual Round! We'll start off with an easy one: in how many distinct ways can the letters in the word *JONMANK* be rearranged?

   (A) 5040   (B) 2520   (C) 1260   (D) 840   (E) NOTA

2. Jonathan loves flying planes. However, due to the fact that he is too young to actually fly a plane, he has settled for creating wooden planes and launching them with rubber bands. In one such endeavor, he attempted to launch his plane in a large room with a volume of 1320 cubic feet. Let it also be noted that the dimensions of the room form an arithmetic sequence with a common difference of 1. If we assume that the plane may only travel in a straight line, then the longest distance the plane may travel can be given by $\sqrt{a}$, What is the value of the product of the digits of a?

   (A) 28   (B) 60   (C) 90   (D) 32   (E) NOTA

3. Dylan and Donald are the leaders of the No-Sleep Gang. All members of the club truly need more sleep, but that's a discussion for another day. Assuming Dylan wakes up every day at 3:20 PM and Donald goes to bed at 1:40 AM, then what is the absolute difference between the **larger** angle formed by the hands of the clock at Dylan's wake-up time and the **smaller** angle formed at Donald's bedtime?

   (A) 150   (B) 170   (C) 180   (D) 216   (E) NOTA

4. Himank, while writing this test, gets really hungry. He decides to drive to Waffle House even though it's 1 in the morning (he should seriously consider joining the No-Sleep Gang). However, his luck turns out to be remarkably bad when Karthik threatens to throw a tree onto Himank's car if he cannot answer the following question: if Karthik rolls a dice twice, what is the probability that the sum of the two numbers he gets is over 7? Solve the question correctly so that Himank can live to write tests another day!

   (A) $\dfrac{5}{12}$ (B) $\dfrac{7}{12}$ (C) $\dfrac{11}{18}$ (D) $\dfrac{7}{18}$ (E) NOTA

5. Tanmay Haldiya's favorite movie is 3 Idiots. He has watched it so many times that he remembers every single line. Unsurprisingly, he loves calling people idiots, especially Anish and Viswa. One day, Anish and Viswa are fed up with Tanmay's insults and decide to throw a question at him. They tell him to find the greatest power 4 that is a factor of $3^{15} + 3^{11} + 3^6 + 1$. Solve the question for Tanmay so that he doesn't get a taste of his own medicine. (Hint: Rewrite 3^{11} and 3^6 as $3 \cdot 3^{10}$ and $3 \cdot 3^5$, respectively.)

   (A) 16   (B) 64   (C) 256   (D) 1024   (E) NOTA

6. Expand the expression $(x + 4)^2(y + 5)^2$. (Hint: the answer is not E.)

   (A) $x^2y^2 + 8x^2y + 10xy^2 + 16x^2 + 80xy + 25y^2 + 160x + 200y + 400$
   (B) $x^2y^2 + 8x^2y + 8xy^2 + 16x^2 + 64xy + 16y^2 + 128x + 128y + 256$
   (C) $x^2y^2 + 10x^2y + 8xy^2 + 25x^2 + 80xy + 16y^2 + 200x + 160y + 400$
   (D) $x^2y^2 + 10x^2y + 10xy^2 + 25x^2 + 100xy + 25y^2 + 250x + 250y + 625$
   (E) NOTA

7. Sukeerth, as the Theta Captain of Rickards, has a lot of responsibilities that he must live up to. However, he's being a bit lazy, which enrages Ms Cross. She threatens to remove him as Theta Captain if he cannot solve the following question: the number of questions on this test divided by 3, when raised to the 4th power contains how many 0s? Help Sukeerth save face!

   (A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

8. Karthik enjoys consuming geometry problems at any hour of the day. He holds geometry problems in such high regard that he consumes 400 of them in one sitting of half an hour, but that's irrelevant here. On one such sitting, he is struck with the following question: What is the area of the rhombus shown below?
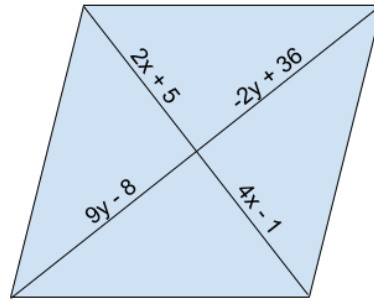

(A) 616   (B) 626   (C) 636   (D) 646   (E) NOTA

9. Dr. Kurian, in contrast, enjoys solving geometry problems. Consider the figure to the left below, depicting sphere A. Inside of sphere A exist 3 more spheres suspended inside of one another, labeled B, C and D, depicted in the diagram to the right below. If sphere D has a radius of 1, sphere C has a radius of 2, sphere B has a radius of 3, and sphere A has a radius of 4, then what is the ratio of the volume outside sphere D but inside sphere C to the volume outside sphere B but inside sphere A? (Hint: the volume of a sphere formula is $\dfrac{4\pi r^3}{3}$, where r is the radius of the sphere.)
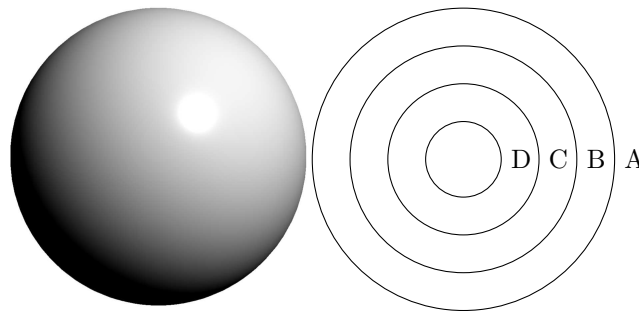

(A) $\dfrac{28}{37}$ (B) $\dfrac{37}{28}$ (C) $\dfrac{1}{8}$ (D) $\dfrac{1}{27}$ (E) NOTA

10. Gollum from *The Hobbit* loves riddles. He asks you the following: "From a group of 7 distinct pieces of gold and 6 distinct pieces of diamond, 5 pieces are to be selected to form a group that contains at least 3 pieces of gold. In how many ways can this be done?"

(A) 91   (B) 756   (C) 231   (D) 735   (E) NOTA

11. Shubham is an avid swimmer. His dream is to swim for the United States at the Olympics some day, and has already started preparing. He notes that an Olympic swimming pool has a length of 50 yards. If Shubham can swim at 8 feet per second, then what is the lowest time in seconds it would take him to make 4 laps? (Assume that a lap is when Shubham swims from one end to the pool to the other and back again.)

(A) 75   (B) 50   (C) 25   (D) 150   (E) NOTA

12. Solve the inequality: $|x - 10| > 3$.

(A) $x \leq -13$ or $x \geq -7$ (B) $x \leq 7$ or $x \geq 13$ (C) $x < 7$ or $x > 13$ (D) $x < -13$ or $x > -7$ (E) NOTA

13. After a long day of AP Exams, Tanmay wants a drink of water before he heads home. Unfortunately, his water bottle is empty, so he needs to head to a river. This scenario is shown below. If Tanmay is standing at the point $(3, 4)$, the river flows along the line $y = 7$, and his home is at the point $(15, -6)$, what is the shortest distance Tanmay must travel so he can get home having quenched his thirst?
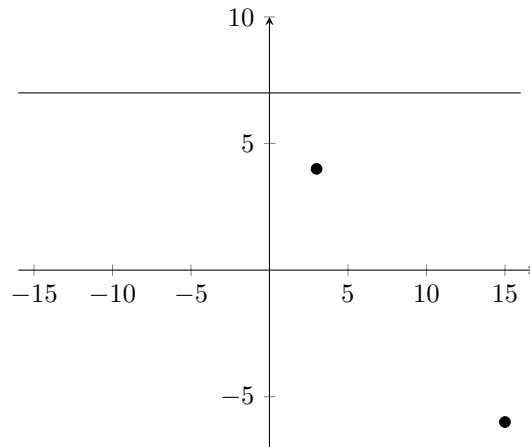

(A) 20   (B) $2\sqrt{61}$ (C) 28   (D) $3\sqrt{17} + 13$ (E) NOTA

14. Himank and Sukeerth are racing! However, Himank leaves Tallahassee at 9 AM and drives on the interstate at 60 miles per hour. In a desperate attempt to catch up, Sukeerth leaves Tallahassee at 10 AM and drives at 70 miles per hour. How long will it take Sukeerth to catch up to Himank?

(A) 5 hours   (B) 6 minutes   (C) 7 hours   (D) 8 hours   (E) NOTA

15. Sagar loves coffee. Every morning, he heads to Starbucks to pick up a vanilla latte along with a lemon cookie, which makes up his entire breakfast. In a vanilla latte, coffee A worth 1.05 dollars per pound is mixed with coffee B worth 85 cents per pound to obtain 20 pounds of a mixture worth 90 cents per pound. How many pounds of each type are used?

(A) 15 lbs. A, 5 lbs. B   (B) 15 lbs. B, 5 lbs. A   (C) 10 lbs. A, 10 lbs. B   (D) 20 lbs. B, 0 lbs. A   (E) NOTA

16. Kevin truly loves math. If given the choice to do math forever, he would take it instantly. However, conversely, he truly hates geometry problems. If given the choice to end geometry forever, he would take it instantly. Sure enough, he was recently forced to confront the conundrum placed before you: Consider right $\triangle ABC$. If $AC = BC$, and $AC = 4$, then what is the sum of the area and perimeter of $\triangle ABC$? Help Kevin retain his love for math by solving the question!
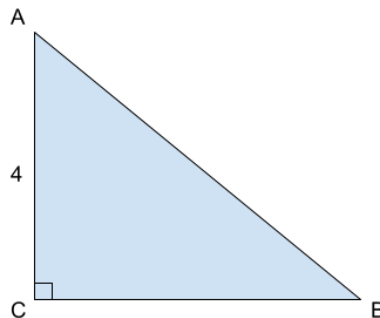
(A) 8   (B) $8 + 4\sqrt{2}$ (C) $16 + 4\sqrt{2}$ (D) $64 + 32\sqrt{2}$ (E) NOTA

17. Amogh, Bilal, Cannon, Donald, Eric, and Farzan are having a party. To celebrate, everyone gives each other a handshake. How many distinct handshakes can ensue?

    (A) 36   (B) 18   (C) 30   (D) 15   (E) NOTA

18. Srikar wishes to be the best at math in the nation. He has a long way to go, but has already begun preparations for it. However, he is stumped with the following question: Mckayleigh flips a coin 4 times. What is the probability that she gets exactly 3 heads? Solve the question for Srikar so he can get better at math!

    (A) $\dfrac{1}{4}$ (B) $\dfrac{1}{16}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{2}{3}$ (E) NOTA

19. Srikar, showcasing his brilliance, realizes that he can't do math forever and decides to become a firefighter. One day, on the job, the Rickards Fire Department gets a call. Srikar slides down the fireman's pole, but suddenly realizes that he is travelling diagonally. Applying his mathematical skill, he realizes that he is travelling from the point $(16, 31)$ to the point $(9, 17)$. The equation of the pole in slope-intercept form can be given as $y = bx + m$, where b and m are the slope and y-intercept of the equation, respectively. Find the sum $b + m$.

    (A) 1   (B) -1   (C) 2   (D) -2   (E) NOTA

20. In a regular polygon, one may define the *apothem* to be the distance from the center of the polygon to the midpoint of one of the polygon's sides. If the apothem of the regular dodecagon shown below is 20, and its side length is 8, then what is the area of the dodecagon? (Hint: draw the apothems and construct triangles).
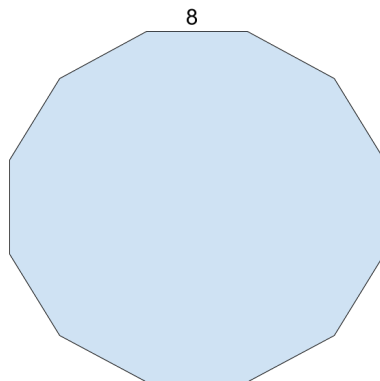
    (A) 960   (B) $384\sqrt{6}$ (C) 1920   (D) $768\sqrt{6}$ (E) NOTA

21. Eric loves to add and multiply. He defines the operation $x \oplus y$ as $\dfrac{xy}{x + y}$ and the operation $x \otimes y$ as $\dfrac{xy}{0.75x + y}$. With this information, evaluate $4 \otimes (2 \oplus 2)$. Express your answer as a decimal to the nearest tenth.

    (A) 0.9   (B) 1.1   (C) 1.0   (D) 1.2   (E) NOTA

22. Himank presents you with the following system of equations:

$$3x + 4y + 5z = 20$$
$$4x + 5y + 3z = 25$$
$$5x + 3y + 4z = 30$$

    Find the value of $x + y + z$. Express your answer as a decimal to the nearest hundredth.

    (A) $\dfrac{25}{4}$ (B) 6.25   (C) $\dfrac{75}{12}$ (D) 6.3   (E) NOTA

23. Himank congratulates you for making it to the last stretch of the test! To reward you for this, he decides to give you a question about progressions. There exists an arithmetic sequence that follows the progression $2, 9, 16, 23, 30, \ldots, 65$. Let the sum of this sequence be x. There also exists a geometric sequence that follows the progression $1, 4, 16, 64, \ldots$ and extends forever. Let the 6th term of this second sequence be y. Find the sum $x + y$.

    (A) 335　　　　　(B) 689　　　　　(C) 1024　　　　　(D) 1359　　　　　(E) NOTA

24. Jonathan loves tricky questions. He gives the following conundrum to you: Let

$$(M - Z)(M - Y)(M - X)(M - W) \ldots (M - A) = x,$$

    where $A, B, C, D, \ldots, Z$ all represent distinct, positive integers. What is the value of x?

   (A) 1　　　　　(B) 2　　　　　(C) 3　　　　　(D) 4　　　　　(E) NOTA

25. Suhas and Vibav love the song 777 by Bruno Mars and Anderson .Paak. They also love multiples of 7 that's digits add up to 10. With this, they pose the following question to you: Consider the equation $7^7 + 7^7 + 7^7 + 7^7 + 7^7 + 7^7 + 7^7 = 343^x$. If $x = \dfrac{m}{n}$ where m and n are positive integers, and $\dfrac{m}{n}$ is fully simplified, then what is the value of $m + n$?

    (A) $\dfrac{49}{3}$　　　　　(B) 52　　　　　(C) 11　　　　　(D) $\dfrac{8}{3}$　　　　　(E) NOTA

26. In similar regard, Nitish loves multiples of 10. However, he hates the number 2022 and its influence on any multiples of 10. He asks you the following: If $10^{2022} - 2022$ is written as an integer in decimal form, what is the sum of its digits?

    (A) 18184　　　　　(B) 18193　　　　　(C) 18202　　　　　(D) 18209　　　　　(E) NOTA

27. Tanmay presents you with the following equations, A, B and C graphed on a Cartesian plane. Which of these equations show growth of any form?
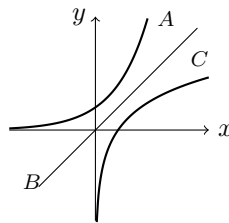
    (A) A　　　　　(B) B　　　　　(C) C　　　　　(D) A, B, C　　　　　(E) NOTA

28. If Himank painted the outside of a $5 \times 5 \times 5$ cube made out of $1 \times 1 \times 1$ unit cubes, how many of the unit cubes would have paint on exactly 3 sides?

    (A) 8　　　　　(B) 6　　　　　(C) 4　　　　　(D) 2　　　　　(E) NOTA

29. Consider a 4×4 grid of dots. Let the number of squares that have all 4 vertices on these dots be x. Let the sum of the digits of 10! be y. Find the sum $x + y$.

    (A) 41　　　　　(B) 43　　　　　(C) 45　　　　　(D) 47　　　　　(E) NOTA

30. How much dirt is in an ellipsoid-shaped hole in the ground with radii $3, 4$ and $\dfrac{5}{\pi}$?

    (A) 80　　　　　(B) 80π　　　　　(C) 0　　　　　(D) Indeterminable　　　　　(E) NOTA